COMMON CORE State Standards

DECONSTRUCTED for CLASSROOM IMPACT

THIRD GRADE
MATHEMATICS

The COMMON CORE Institute

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Introduction

The Common Core Institute is pleased to offer this grade-level tool for educators who are teaching with the Common Core State Standards.

The Common Core Standards Deconstructed for Classroom Impact is designed for educators by educators as a two-pronged resource and tool 1) to help educators increase their depth of understanding of the Common Core Standards and 2) to enable teachers to plan College & Career Ready curriculum and classroom instruction that promotes inquiry and higher levels of cognitive demand.

What we have done is not all new. This work is a purposeful and thoughtful compilation of preexisting materials in the public domain, state department of education websites, and original work by the Center for College & Career Readiness. Among the works that have been compiled and/or referenced are the following: Common Core State Standards for Mathematics and the Appendix from the Common Core State Standards Initiative; Learning Progressions from The University of Arizona's Institute for Mathematics and Education, chaired by Dr. William McCallum; the Arizona Academic Content Standards; the North Carolina Instructional Support Tools; and numerous math practitioners currently in the classroom.

We hope you will find the concentrated and consolidated resource of value in your own planning. We also hope you will use this resource to facilitate discussion with your colleagues and, perhaps, as a lever to help assess targeted professional learning opportunities.

Understanding the Organization

The **Overview** acts as a quick-reference table of contents as it shows you each of the domains and related clusters covered in this specific grade-level booklet. This can help serve as a reminder of what clusters are part of which domains and can reinforce the specific domains for each grade level.

Math Fluency Standards	
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20
3	Multiply/divide within 100
4	Add/subtract within 1,000,000
5	Multi-digit multiplication
6	Multi-digit division
7	Solve
8	Solve simple 2 x 2 systems by inspection

Key Changes identifies what has been moved to and what has been moved from this particular grade level, as appropriate. This section also includes **Critical Areas of Focus**, which is designed to help you begin to approach how to examine your curriculum, resources, and instructional practices. A review of the **Critical Areas of Focus** might enable you to target specific areas of professional learning to refresh, as needed.

For each domain is the domain itself and the associated clusters. Within each domain are sections for each of the associated clusters. The cluster-specific content can take you to a deeper level of understanding. Perhaps most importantly, we include here the **Learning Progressions**. The **Learning Progressions** provide context for the current domain and its related standards. For any grade except Kindergarten, you will see the domain-specific standards for the current

grade in the center column. To the left are the domain-specific standards for the preceding grade and to the right are the domain-specific standards for the following grade. Combined with the **Critical Areas of Focus**, these **Learning Progressions** can assist you in focusing your planning.

For each cluster, we have included four key sections: Description, Big Idea, Academic Vocabulary, and Deconstructed Standard.

The cluster **Description** offers clarifying information, but also points to the **Big Idea** that can help you focus on that which is most important for this cluster within this domain. The **Academic Vocabulary** is derived from the cluster description and serves to remind you of potential challenges or barriers for your students.

Each standard specific to that cluster has been deconstructed. There **Deconstructed Standard** for each standard specific to that cluster and each **Deconstructed Standard** has its own subsections, which can provide you with additional guidance and insight as you plan. Note the deconstruction drills down to the sub-standards when appropriate. These subsections are:

- Standard Statement
- Standard Description
- Essential Question(s)
- Mathematical Practice(s)
- DOK Range Target for Learning and Assessment
- Learning Expectations
- Explanations and Examples

As noted, first are the **Standard Statement** and **Standard Description**, which are followed by the **Essential Question(s)** and the associated **Mathematical Practices**. The **Essential Question(s)** amplify the **Big Idea**, with the intent of taking you to a deeper level of understanding; they may also provide additional context for the **Academic Vocabulary**.

The **DOK Range Target for Learning and Assessment** remind you of the targeted level of cognitive demand. The **Learning Expectations** correlate to the DOK and express the student learning targets for student proficiency for KNOW, THINK, and DO, as appropriate. In some instances, there may be no learning targets for student proficiency for one or more of KNOW, THINK or DO. The learning targets are expressions of the deconstruction of the Standard as well as the alignment of the DOK with appropriate consideration of the Essential Questions.

The last subsection of the **Deconstructed Standard** includes **Explanations and Examples.** This subsection might be quite lengthy as it can include additional context for the standard itself as well as examples of what student work and student learning could look like. **Explanations and Examples** may offer ideas for instructional practice and lesson plans.

Standards for Mathematical Practice in 3rd Grade

Each of the explanations below articulates some of the knowledge and skills expected of students to demonstrate grade-level mathematical proficiency.

PRACTICE	EXPLANATION
Make sense and persevere in problem solving.	Students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it; they may use concrete objects or pictures to help them conceptualize and solve problems. It becomes more routine for students to check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.
Reason abstractly and quantitatively.	Students recognize that a number represents a specific quantity; they connect the quantity to written symbols and can create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
Construct viable arguments and critique the reasoning of others.	Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills in discussion the teacher facilities by asking questions such as "How did you get that?" and "Why is that true?". They explain their thinking to others and respond to others' thinking.
Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), and graphical representations. Students have extensive opportunities to generate various mathematical representations and to solve both equations and story problems; they are able to explain connections between representations as well as between representations and equations. Students are able to evaluate their results in the context of the situation and reflect on whether the results make sense.
Use appropriate tools strategically.	Students consider the available tools (including estimation) when solving a problem and decide when certain tools might be helpful.
Attend to precision.	Students develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose.
Look for and make use of structure.	Students look closely to discover a pattern or structure, such as commutative and distributive properties.
Look for and express regularity in repeated reasoning.	Students begin to look for regularity in problem structures when solving mathematical tasks. Further, students use repeated reasoning while solving a task with multiple correct answers.
Look for an express regularity in repeated reasoning.	Students should notice repetitive actions in computation and look for more shortcut methods, recognizing, for instance, how to apply the commutative and distributive properties.

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

OVERVIEW

Operations and Algebraic Thinking (OA)

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten (NBT)

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations – Fractions (NF)

- Develop understanding of fractions as numbers.

Measurement and Data (MD)

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry (G)

- Reason with shapes and their attributes.

Mathematical Practices (MP)

MB 1. Make sense of problems and persevere in solving them.

MB 2. Reason abstractly and quantitatively.

MB 3. Construct viable arguments and critique the reasoning of others.

MB 4. Model with mathematics.

MB 5. Use appropriate tools strategically.

MB 6. Attend to precision.

MB 7. Look for and make use of structure.

MB 8. Look for and express regularity in repeated reasoning.

KEY CHANGES

NEW TO THIRD GRADE	▪ **Area and perimeter (3.MD.5, 3.MD.6, 3.MD.7)**
MOVED FROM THIRD GRADE	▪ **Permutation and combinations (4.02, 4.03)** ▪ **Rectangular Coordinate System (3.02)** ▪ **Circle graphs (4.01)**

KEY CHANGES

CRITICAL AREAS OF FOCUS

1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 100.

- Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1).

- Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Developing understanding of the structure of rectangular arrays and of area.

- Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

4. Describing and analyzing two-dimensional shapes.

- Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

DOMAIN:

OPERATIONS AND ALGEBRAIC THINKING (OA)

THIRD GRADE

MATHEMATICS

DOMAIN	**Operations and Algebraic Thinking (OA)**
CLUSTERS	**1. Represent and solve problems involving multiplication and division.** **2. Understand properties of multiplication and the relationship between multiplication and division.** **3. Multiply and divide within 100.** **4. Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

OPERATIONS AND ALGEBRAIC THINKING (OA)

SECOND	THIRD	FOURTH
Early Equations and Expressions		
Section 1: Exploring arithmetic and geometric patterns/sequences	Section 1: Exploring arithmetic and geometric patterns/sequences	Section 1: Exploring arithmetic and geometric patterns/sequences
	3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.	4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.
Section 2: Exploring Equations	Section 2: Exploring Equations	Section 2: Exploring Equations
	3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.	
	3.OA.8 Solve two-step word problems using the four operations (restricted to whole numbers) and apply rules for order of operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
Multiplication and Division		
Section 1: Understanding and Relating Multiplication and Division Operations	Section 1: Understanding and Relating Multiplication and Division Operations	Section 1: Understanding and Relating Multiplication and Division Operations
2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	1.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7.	4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
	3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.	4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
	3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	
	OA.6 Understand division as an unknown-factor problem.	

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

OPERATIONS AND ALGEBRAIC THINKING (OA)		
SECOND	**THIRD**	**FOURTH**
Section 2: Multiplication and Division Properties and Facts	Section 2: Multiplication and Division Properties and Facts	Section 2: Multiplication and Division Properties and Facts
	3.OA.5 Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (commutative property of multiplication). 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (associative property of multiplication). Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (distributive property).	
	3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Third Grade, know from memory all products of two one-digit numbers.	
Section 3: Factors and Multiples	Section 3: Factors and Multiples	Section 3: Factors and Multiples
		4.OA.4 Find all factor pairs for a whole number in the range 1 - 100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1 - 100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1 - 100 is prime or composite.
		Students distinguish the terms "factors" and "multiples" in multiplication.
		4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
		4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
		4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Section 5: Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)	Section 5: Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)	Section 5: Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)
		NF.4.a Understand a fraction a/b as a multiple of 1/b.
		4.NF.4.b Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
		NF.4.c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Source: turnonccmath.net, NC State University College of Education

CLUSTER :	**1. Represent and solve problems involving multiplication and division.**
	Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. An important component of solving problems involving addition and subtraction is the ability to recognize that any given group of objects (up to 10) can be separated into sub groups in multiple ways and remain equivalent in amount to the original group (e.g., A set of 6 cubes can be separated into a set of 2 cubes and a set of 4 cubes and remain 6 total cubes).
BIG IDEA:	Multiplication and division are related operations
ACADEMIC VOCABULARY:	products, groups of, quotients, partitioned equally, multiplication, division, equal groups, group size, arrays, equations, unknown

STANDARD AND DECONSTRUCTION

3.OA.1	**Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 x 7.**
DESCRIPTION	This standard interprets products of whole numbers.

PROBLEM TYPE: COMPARE

Difference Unknown:	***Bigger Unknown:***
Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3, 5 x 3 = 15. Describe another situation where there would be 5 groups of 3 or 5 x 3.	Sonya earns $7 a week pulling weeds. After 5 weeks of work, how much has Sonya worked? Write an equation and find the answer. Describe another situation that would match 7x5.

ESSENTIAL QUESTION(S) FOR THE STANDARD	How can I find the total number of objects in equal groups?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Find the product of multiple groups of objects.	Interpret products of whole numbers as a total number of objects in a number of groups.	

EXPLANATIONS AND EXAMPLES

Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers, and equations. Then, given a multiplication expression (e.g., 5 x 6) students interpret the expression using a multiplication context (see Table 2). They should begin to use the terms "factor" and "product" as they describe multiplication.

Students may use interactive whiteboards to create digital models.

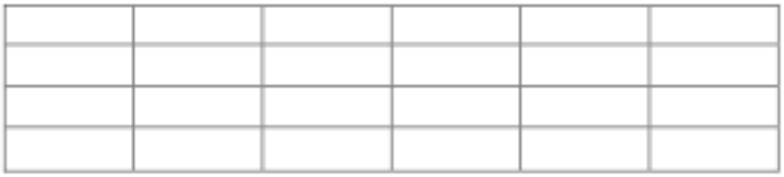
STANDARD AND DECONSTRUCTION

3.OA.2	**Interpret whole-number quotients of whole numbers, e.g. interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.**
DESCRIPTION	This standard focuses on two distinct models of division: partition models and measurement (repeated subtraction) models. Partition models provide students with a total number and the number of groups. These models focus on the question, "How many objects are in each group so that the groups are equal?" A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag? Measurement (repeated subtraction) models provide students with a total number and the number of objects in each group. These models focus on the question, "How many equal groups can you make?" A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill? 
ESSENTIAL QUESTION(S) FOR THE STANDARD	When you divide what do you find?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Know what the numbers in a division problem represent.	Explain what division means and how it relates to equal shares Interpret quotients as the number of shares or the number of groups when a set of objects is divided equally.	

EXPLANATIONS AND EXAMPLES	Students recognize the operation of division in two different types of situations. One situation requires determining how many groups and the other situation requires sharing (determining how many in each group). Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor). To develop this understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. Given a division expression (e.g., 24 ÷ 6) students interpret the expression in contexts that require both interpretations of division (see Table 2). Students may use interactive whiteboards to create digital models.

STANDARD AND DECONSTRUCTION

3.OA.3	**Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.**
DESCRIPTION	This standard references various problem solving context and strategies that students are expected to use while solving word problems involving multiplication & division. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many cookies does each person receive? (4 x 9 = 36, 36 ÷ 6 = 6). The glossary (see Table 2, page 89; the table is included at the end of this document for your reference) gives examples of a variety of problem solving contexts in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the different problem structures. Examples of multiplication: There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there? This task can be solved by drawing an array by putting 6 desks in each row. This is an array model.  This task can also be solved by drawing pictures of equal groups; 4 groups of 6 equals 24 objects.  A student can also reason through the problem mentally or verbally, "I know 6 and 6 are 12. 12 and 12 are 24. Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom." A number line could also be used to show equal jumps. Students in Third Grade should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables). Letters are also introduced to represent unknowns in Third Grade.
ESSENTIAL QUESTION(S) FOR THE STANDARD	How do I know when to use multiplication or division to solve a problem?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

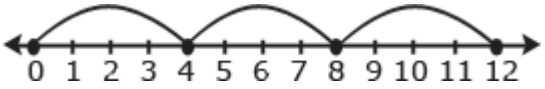
Instructional Targets	Know: Concepts/Skills	Think	Do
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Multiply and divide within 100.	Solve word problems in situations involving equal groups, arrays, and measurement quantities Represent a word problem using a picture, and an equation with a symbol for the unknown number.	

EXPLANATIONS AND EXAMPLES

Students use a variety of representations for creating and solving one-step word problems, e.g., numbers, words, pictures, physical objects, or equations. They use multiplication and division of whole numbers up to 10 x10. Students explain their thinking, show their work by using at least one representation, and verify that their answer is reasonable.

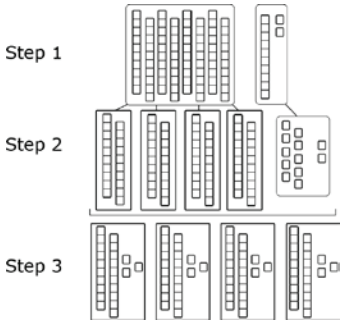
Word problems may be represented in multiple ways:

- Equations: 3 x 4 = ?, 4 x 3 = ?, 12 ÷ 4 = ? and 12 ÷ 3 = ?
- Array:
- Equal groups
- Repeated addition: 4 + 4 + 4 or repeated subtraction
- Three equal jumps forward from 0 on the number line to 12 or three equal jumps backwards from 12 to 0



Examples of division problems:

- Determining the number of objects in each share (partitive division, where the size of the groups is unknown):
- The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?



- Determining the number of shares (measurement division, where the number of groups is unknown)

Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	24-4= 20	20-4= 16	16-4= 12	12-4= 8	8-4= 4	4-4= 0

Solution: The bananas will last for 6 days.

Students may use interactive whiteboards to show work and justify their thinking.

STANDARD AND DECONSTRUCTION

3.OA.4	**Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = ? ÷ 3, 6 x 6 = ?.**
DESCRIPTION	This standard refers to the glossary (see Table 2, page 89; the table is included at the end of this document for your reference) and equations for the different types of multiplication and division problem structures. The easiest problem structures include Unknown Product (3 x 6 = ? or 18 ÷ 3 = 6). The more difficult problem structures include Group Size Unknown (3 x ? = 18 or 18 ÷ 3 = 6) or Number of Groups Unknown (? x 6 = 18, 18 ÷ 6 = 3). The focus of 3.OA.4 extends beyond the traditional notion of fact families by having students explore the inverse relationship of multiplication and division.
ESSENTIAL QUESTION(S) FOR THE STANDARD	In what ways can multiplication and division affect numbers? How does multiplication and division offer efficient ways to get an answer?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.2. Reason abstractly and quantitatively. 3.MP.6. Attend to precision. 3.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Multiply and divide within 100.	Determine which operation (multiplication or division) is needed to determine the unknown whole number. Solve to find the unknown whole number in a multiplication or division equation.	

EXPLANATIONS AND EXAMPLES

This standard is strongly connected to 3.AO.3 when students solve problems and determine unknowns in equations. Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation.

Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 4 x ? = 40, they might think:

- 4 groups of some number is the same as 40
- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Examples:

- Solve the equations below:

24 = ? x 6

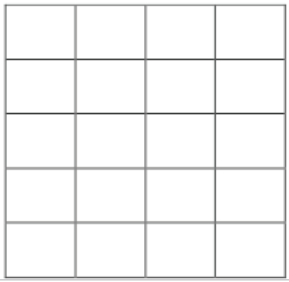
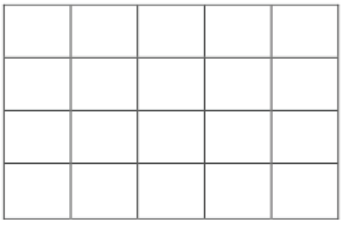
- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether?
  3 x 4 = m

$$72 \div \Delta = 9$$

Students may use interactive whiteboards to create digital models to explain and justify their thinking.

CLUSTER:	**2. Represent and solve problems involving multiplication and division.**
	Understand properties of multiplication and the relationship between multiplication and division.
BIG IDEA:	Multiplication and division are related operations.
ACADEMIC VOCABULARY:	operation, multiply, divide, factor, product, quotient, strategies, properties (rules about how numbers work)

STANDARD AND DECONSTRUCTION

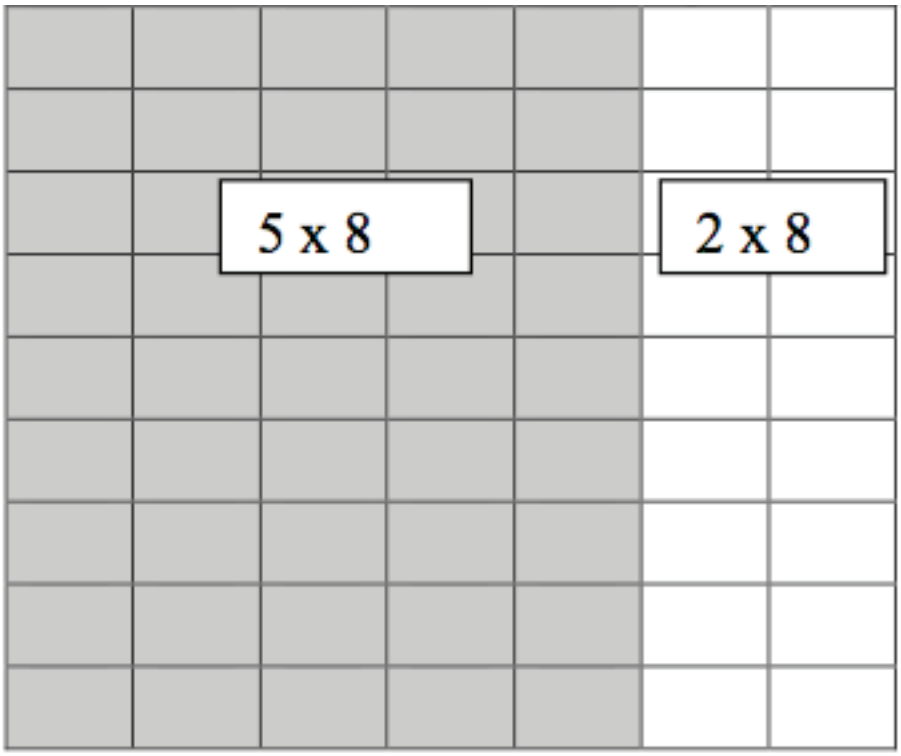
3.OA.5	**Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (commutative property of multiplication). 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (associative property of multiplication). Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (distributive property).**
DESCRIPTION	This standard references properties (rules about how numbers work) of multiplication. This extends past previous expectations, in which students were asked to identify properties. While students DO NOT need to use the formal terms of these properties, students must understand that properties are rules about how numbers work, and they need to be flexibly and fluently applying the rules in various situtations. Students represent expressions using various objects, pictures, words, and symbols to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication. The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies 7 x 5 x 2, a student could rearrange the numbers to first multiply 5 x 2 = 10 and then multiply 10 x 7 = 70. The commutative property (order property) states that the order of numbers does not matter when you are adding or multiplying numbers. For example, if a student knows that 5 x 4 = 20, then they also know that 4 x 5 = 20. The array below could be described as a 5 x 4 array for 5 columns and 4 rows, or a 4 x 5 array for 4 rows and 5 columns. There is no fixed way to write the dimensions of an array as rows x columns or columns x rows. Students should have flexibility in being able to describe both dimensions of an array. Example: 4 x 5 or 5 x 4   4 x 5 or 5 x 4

DESCRIPTION (CONTINUED)

Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. Students would be using mental math to determine a product. Here are ways that students could use the distributive property to determine the product of 7 x 6. Again, students should use the distributive property, but can refer to this in informal language such as "breaking numbers apart."

Student 1	Student 2	Student 3
7 x 6	7 x 6	7 x 6
7 x 5 = 35	7 x 3 = 21	5 x 6 = 30
7 x 1 = 7	7 x 3 = 21	2 x 6 = 12
35 + 7 = 42	21 + 21 = 42	30 + 12 = 42

Another example of the distributive property helps students determine the products and factors of problems by breaking numbers apart. For example, for the problem 7 x 8 = ?, students can decompose the 7 into a 5 and 2, and reach the answer by multiplying 5 x 8 = 40 and 2 x 8 =16 and adding the two products (40 +16 = 56).



ESSENTIAL QUESTION(S) FOR THE STANDARD

What are the properties of multiplication and division?

MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.7. Look for and make use of structure. 3.MP.8. Look for and express regularity in repeated reasoning.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Multiply and divide within 100.	Explain how the properties of operations work. Apply properties of operations as strategies to multiply and divide.	

EXPLANATIONS AND EXAMPLES

Students represent expressions using various objects, pictures, words, and symbols to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.

Models help build understanding of the commutative property:

Example: 3 x 6 = 6 x 3

In the following diagram it may not be obvious that 3 groups of 6 is the same as 6 groups of 3. A student may need to count to verify this.

 is the same quantity as 

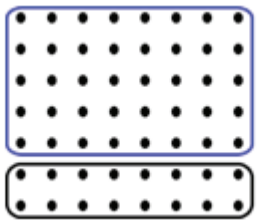
Example: 4 x 3 = 3 x 4

An array explicitly demonstrates the concept of the commutative property.

4 rows of 3 or 4 x 3 3 rows of 4 or 3 x 4

Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. Students should learn that they can decompose either of the factors. It is important to note that the students may record their thinking in different ways.

5 x 8 = 40
2 x 8 = 16
56



7 x 4 = 28
7 x 4 = 28
56

EXPLANATIONS AND EXAMPLES

To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.

- 0 x 7 = 7 x 0 = 0 (Zero Property of Multiplication)
- 1 x 9 = 9 x 1 = 9 (Multiplicative Identity Property of 1)
- 3 x 6 = 6 x 3 (Commutative Property)
- 8 ÷ 2 = 2 ÷ 8 (Students determine only that these are not equal)
- 2 x 3 x 5 = 6 x 5
- 10 x 2 < 5 x 2 x 2
- 2 x 3 x 5 = 10 x 3
- 0 x 6 > 3 x 0 x 2

STANDARD AND DECONSTRUCTION

3.OA.6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
DESCRIPTION	This standard refers to the glossary (see Table 2, page 89; the table is included at the end of this document for your reference) and the various problem structures. Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems. Example: A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning. Example: Sarah did not know the answer to 63 divided by 7. Are each of the following was an appropriate way for Sarah to think about the problem? Explain why or why not with a picture or words for each one. I know that 7 x 9=63, so 63 divided by 7 must be 9. I know that 7 x 10 = 70. If I take away a group of 7 ,that means that I have 7 x 9 = 63. So 63 divided by 7 is 9. I know that 7 x 5 is 35. 63 minus 35 is 28. I know that 7 x 4 = 28. So if I add 7 x 5 and 7 x 4 I get 63. That means that 7 x 9 is 63, or 63 divided by 7 is 9.
ESSENTIAL QUESTION(S) FOR THE STANDARD	In what ways can division affect numbers?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	Know: Concepts/Skills	Think	Do
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Identify the multiplication problem as related to the division problem. Identify the unknown factor in the related multiplication problem.	Recognize multiplication and division as related operations and explain how they are related. Use multiplication to solve division problems.	

EXPLANATIONS AND EXAMPLES

Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Examples:

- 3 x 5 = 15 5 x 3 = 15
- 15 ÷ 3 = 5 15 ÷ 5 = 3

Students use their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 32 ÷ | = 4, students may think:

- 4 groups of some number is the same as 32
- 4 times some number is the same as 32
- I know that 4 groups of 8 is 32 so the unknown number is 8
- The missing factor is 8 because 4 times 8 is 32

Equations in the form of a ÷ b = c and c = a ÷ b need to be used interchangeably, with the unknown in different positions.

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

CLUSTER:	**3. Represent and solve problems involving multiplication and division.**
	Multiply and divide within 100.
BIG IDEA:	Multiplication and division are related operations.
ACADEMIC VOCABULARY:	operation, multiply, divide, factor, product, quotient, unknown, strategies, reasonableness, mental computation, property

STANDARD AND DECONSTRUCTION

3.OA.7	**Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Third Grade, know from memory all products of two one-digit numbers.**
DESCRIPTION	This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). "Know from memory" should not focus only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 x 9). Note that mastering this material, and reaching fluency in single-digit multiplications and related divisions with understanding, may be quite time consuming because there are no general strategies for multiplying or dividing all single-digit numbers as there are for addition and subtraction. Instead, there are many patterns and strategies dependent upon specific numbers. So it is imperative that extra time and support be provided if needed (Progressions for the CCSSM; Operations and Algebraic Thinking, CCSS Writing Team, May 2011, page 22) All of the understandings of multiplication and division situations of the levels of representation and solving, and of patterns need to culminate by the end of Grade 3 in fluent multiplying and dividing of all single-digit numbers and 10. Such fluency may be reached by becoming fluent for each number (e.g., the 2s, the 5s, etc.) and then extending the fluency to several, then all numbers mixed together. Organizing practice so that it focuses most heavily on understood but not yet fluent products and unknown factors can speed learning. To achieve this by the end of Third Grade, students must begin working toward fluency for the easy numbers as early as possible. Because an unknown factor (a division) can be found from the related multiplication, the emphasis at the end of the year is on knowing from memory all products of two one-digit numbers. As should be clear from the foregoing, this isn't a matter of instilling facts divorced from their meanings, but rather the outcome of a carefully designed learning process that heavily involves the interplay of practice and reasoning. All of the work on how different numbers fit with the base-ten numbers culminates in these "just know" products and is necessary for learning products. Fluent dividing for all single-digit numbers, which will combine just knowing from a multiplication patterns and best strategy, is also part of this vital standard (Progressions for the CCSSM; Operations and Algebraic Thinking, CCSS Writing Team, May 2011, page 27).

ESSENTIAL QUESTION(S) FOR THE STANDARD	Which strategy is most efficient when solving multiplication and division problems? Why do we need math facts?
MATHEMATICAL PRACTICE(S)	3.MP.2. Reason abstractly and quantitatively. 3.MP.7. Look for and make use of structure. 3.MP.8. Look for and express regularity in repeated reasoning

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Know from memory all products of two one-digit numbers. Fluently multiply and divide within 100.	Analyze a multiplication or division problem in order to choose an appropriate strategy to fluently multiply or divide within 100.	

EXPLANATIONS AND EXAMPLES

By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Strategies students may use to attain fluency include:

- Multiplication by zeros and ones
- Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
- Tens facts (relating to place value, 5 x 10 is 5 tens or 50)
- Five facts (half of tens)
- Skip counting (counting groups of __ and knowing how many groups have been counted)
- Square numbers (ex: 3 x 3)
- Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3)
- Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6)
- Turn-around facts (Commutative Property)
- Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24)
- Missing factors

General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

CLUSTER:	**4. Represent and solve problems involving multiplication and division.**
	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
BIG IDEA:	An unknown quantity can be determined quickly using the four operations and patterns in arithmetic.
ACADEMIC VOCABULARY:	operation, multiply, divide, factor, product, quotient, subtract, add, addend, sum, difference, equation, unknown, strategies, reasonableness, mental computation, estimation, rounding, patterns, properties (rules about how numbers work)

STANDARD AND DECONSTRUCTION

3.OA.8

Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

DESCRIPTION

Students in Third Grade begin the step to formal algebraic language by using a letter for the unknown quantity in expressions or equations for one and two-step problems. But the symbols of arithmetic, x or. or * for multiplication, and ÷ or / for division, continue to be used in Grades 3, 4, and 5 (Progressions for the CCSSM; Operations and Algebraic Thinking, CCSS Writing Team, May 2011, page 27).

This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related Third Grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100.

This standard calls for students to represent problems using equations with a letter to represent unknown quantities. Example:

Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution (2 x 5 + m = 25).

This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving and then revisit their estimate to check for reasonableness.

Example:

Here are some typical estimation strategies for the problem:

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?

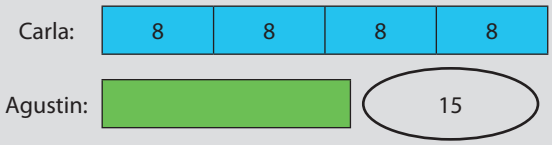
Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

DESCRIPTION

A two-step problem with diagram showing problem situation and equations showing the two parts

Carla has 4 packages of silly brands. Each package has 8 silly bands in it. Agustin is sopposed to get 15 fewer silly bands than Carla. How many silly bands should Agustin get?



C = number of Carla's silly bands
A = number of Agustin's silly bands

$C = 4 \times 8 = 32$

$A + 15 = C$

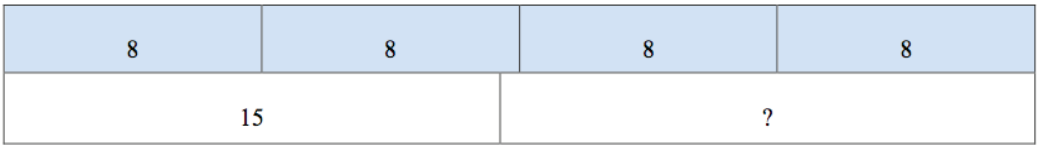
$A + 15 = 32$

$A = 17$

Students may be able to solve this problem without writing such equations

(Progressions for the CCSSM; Operations and Algebraic Thinking, CCSS Writing Team, May 2011, page 28)

In the diagram above, Carla's bands are shown using 4 equal-sized bars that represent 4x8 or 32 bands. Agustin's bands are directly below showing the number Agustin has plus 15 = 32. The diagram can also be drawn like this:



ESSENTIAL QUESTION(S) FOR THE STANDARD

Why is it important to develop an equation to determine an unknown quantity in a two-step word problem? What strategies can be used to solve two–step word problems using the four operations?

MATHEMATICAL PRACTICE(S)

3.MP.1. Make sense of problems and persevere in solving them.

3.MP.2. Reason abstractly and quantitatively.

3.MP.4. Model with mathematics.

3.MP.5. Use appropriate tools strategically.

DOK Range Target for Instruction & Assessment

☒ 1 ☒ 2 ☒ 3 ☐ 4

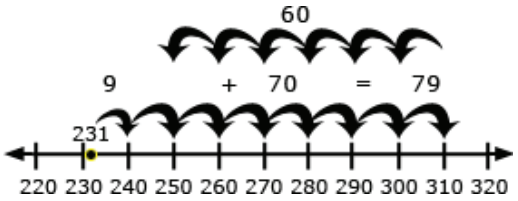
Instructional Targets	Know: Concepts/Skills	Think	Do
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Know the order of operations. Know strategies for estimating and mental computation.	Construct an equation with a letter standing for the unknown quantity. Solve two-step word problems using the four operations. Justify answers to problems using various estimation strategies.	

EXPLANATIONS AND EXAMPLES

Students should be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use.

Examples:

• Jerry earned 231 points at school last week. This week he earned 79 points. If he uses 60 points to earn free time on a computer, how many points will he have left?

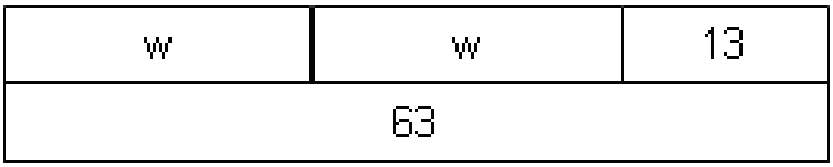


A student may use the number line above to describe his/her thinking, "231 + 9 = 240 so now I need to add 70 more. 240, 250 (10 more), 260 (20 more), 270, 280, 290, 300, 310 (70 more). Now I need to count back 60. 310, 300 (back 10), 290 (back 20), 280, 270, 260, 250 (back 60)."

A student writes the equation, 231 + 79 – 60 = m and uses rounding (230 + 80 – 60) to estimate.

A student writes the equation 231 + 79 – 60 = m and calculates 79-60 = 19 and then calculates 231 + 19 = m.

• The soccer club is going on a trip to the water park. The cost of attending the trip is $63. Included in that price is $13 for lunch and the cost of 2 wristbands, one for the morning and one for the afternoon. Write an equation representing the cost of the field trip and determine the price of one wristband.



The above diagram helps the student write the equation w + w + 13 = 63. Using the diagram, a student might think, "I know that the two wristbands cost $50 ($63-$13) so one wristband costs $25." To check for reasonableness, a student might use front end estimation and say 60-10 = 50 and 50 ÷ 2 = 25.

When students solve word problems, they use various estimation skills which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions.

Estimation strategies include, but are not limited to:

• using benchmark numbers that are easy to compute

• front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts)

• rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding changed the original values)

STANDARD AND DECONSTRUCTION

3.OA.9

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

DESCRIPTION

This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

This standard also mentions identifying patterns related to the properties of operations. Examples:

Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7).

Multiples of even numbers (2, 4, 6, and 8) are always even numbers.

On a multiplication chart, the products in each row and column increase by the same amount (skip counting).

On an addition chart, the sums in each row and column increase by the same amount.

What do you notice about the numbers highlighted in pink in the multiplication table?

Explain a pattern using properties of operations.

Even if the order of the factors is changed, the product remains the same. For example 6 x 5 = 30 and 5 x 6 = 30 (commutative property).

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?
Student: The product will always be an even number.

DESCRIPTION

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What patterns do you notice in this addition table? Explain why the pattern works this way.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	19	11	12	13	14	15	16	17	18	19	20

ESSENTIAL QUESTION(S) FOR THE STANDARD

Where can you see patterns in our world (music, art, architecture, nature, words, numbers)?
How do patterns help us understand our world (music, art, architecture, nature, words, numbers)?

MATHEMATICAL PRACTICE(S)

3.MP.1. Make sense of problems and persevere in solving them.

3.MP.2. Reason abstractly and quantitatively.

3.MP.3. Construct viable arguments and critique the reasoning of others.

3.MP.6. Attend to precision.

3.MP.7. Look for and make use of structure.

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Identify arithmetic patterns.	Explain rules for a pattern using properties of operations. Explain relationships between the numbers in a pattern.	

EXPLANATIONS AND EXAMPLES

Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically. For example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.
- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
•	•	•
•	•	•
•	•	•
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THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

DOMAIN:

NUMBER & OPERATIONS IN BASE TEN (NBT)

THIRD GRADE

MATHEMATICS

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

DOMAIN	**Number and Operations in Base Ten (NBT)**
CLUSTERS	**1. Use place value understanding and properties of operations to perform multi-digit arithmetic.**

NUMBER & OPERATIONS IN BASE TEN (NBT)

SECOND	THIRD	FOURTH
Place Value and Decimals		
Section 2: Three-digit Whole Numbers	Section 2: Three-digit Whole Numbers	Section 2: Three-digit Whole Numbers
2.NBT.1.a 100 can be thought of as a bundle of ten tens, called a "hundred."	NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.	
2.NBT.1.b The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10 - 90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.	
2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.		
NBT.8 Mentally add 10 or 100 to a given number 100 - 900, and mentally subtract 10 or 100 from a given number 100 - 900.		
Section 3: Multi-digit Whole Numbers	Section 3: Multi-digit Whole Numbers	Section 3: Multi-digit Whole Numbers
		4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
		4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
		NBT.3 Use place value understanding to round multi-digit whole numbers to any place.
Section 4: Decimal Numbers, Integer Exponents, and Scientific Notation	Section 4: Decimal Numbers, Integer Exponents, and Scientific Notation	Section 4: Decimal Numbers, Integer Exponents, and Scientific Notation
		NF.6 Use decimal notation for fractions with denominators 10 or 100.
		4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
Addition and Subtraction		
Section 3: Addition and Subtraction Within 1000	Section 3: Addition and Subtraction Within 1000	Section 3: Addition and Subtraction Within 1000
2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

NUMBER & OPERATIONS IN BASE TEN (NBT)		
SECOND	**THIRD**	**FOURTH**
2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.		
2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.		
OA.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.		
2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.		
2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.		
2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.		
Multiplication and Division		
Section 1: Understanding and Relating Multiplication and Division Operations	Section 1: Understanding and Relating Multiplication and Division Operations	Section 1: Understanding and Relating Multiplication and Division Operations
2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7.	4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
	3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.	4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
	3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	
	3.OA.6 Understand division as an unknown-factor problem.	
Section 2: Multiplication and Division Properties and Facts	Section 2: Multiplication and Division Properties and Facts	Section 2: Multiplication and Division Properties and Facts
	3.OA.5 Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known (commutative property of multiplication). If $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$ (associative property of multiplication). Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ (distributive property.)	

NUMBER & OPERATIONS IN BASE TEN (NBT)		
SECOND	**THIRD**	**FOURTH**
	3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	
Section 3: Factors and Multiples	Section 3: Factors and Multiples	Section 3: Factors and Multiples
		4.OA.4 Find all factor pairs for a whole number in the range 1 - 100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1 - 100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1 - 100 is prime or composite. ***Students distinguish the terms "factors" and "multiples" in multiplication.***
		4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
		4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Section 5: Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)	Section 5: Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)	Section 5: Multiplication and Division Problems Involving Non-Whole Rational Number Operators (Fractions)
		4.NF.4.a Understand a fraction a/b as a multiple of 1/b.
		4.NF.4.b Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
		4.NF.4.c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Source: turnonccmath.net, NC State University College of Education

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

CLUSTER:	**1. Use place value understanding and properties of operations to perform multi-digit arithmetic.**
	Multiply and divide within 100.
BIG IDEA:	Deep understanding of place value is needed to use the 4 operations and perform arithmetic problems
ACADEMIC VOCABULARY:	place value, round, addition, add, addend, sum, subtraction, subtract, difference, strategies, (properties) rules about how numbers work.

STANDARD AND DECONSTRUCTION

3.NBT.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
DESCRIPTION	This standard refers to place value understanding, which extends beyond an algorithm or memorized procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding. Mrs. Rutherford drives 158 miles on Saturday and 171 miles on Sunday. She told her husband she estimated how many miles to the nearest 10 before adding the total. She told her sister she estimated to the nearest 100 before adding the total. Which method provided a closer estimate?
ESSENTIAL QUESTION(S) FOR THE STANDARD	How does a digit's position affect its value?
MATHEMATICAL PRACTICE(S)	3.MP.5. Use appropriate tools strategically. 3.MP.7. Look for and make use of structure. 3.MP.8. Look for and express regularity in repeated reasoning.
DOK Range Target for Instruction & Assessment	☒ 1 ☐ 2 ☐ 3 ☐ 4

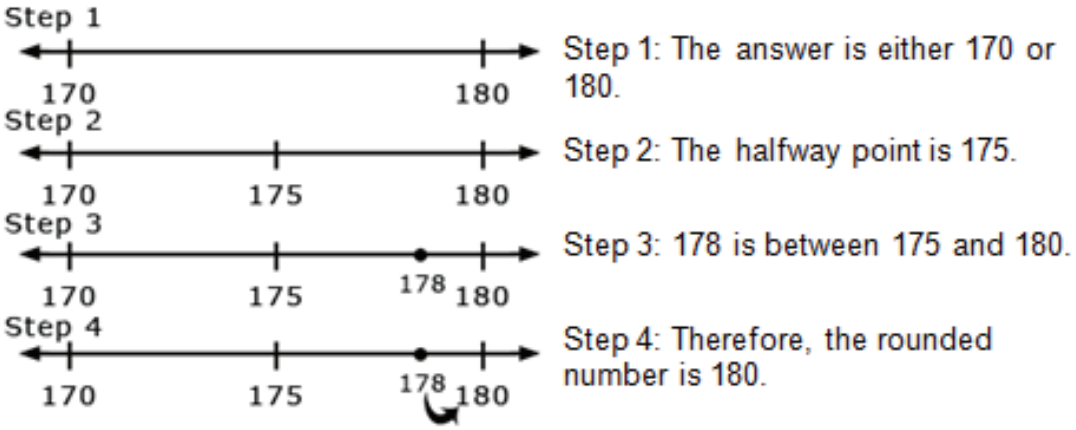
Instructional Targets	Know: Concepts/Skills	Think	Do
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Define round or rounding in relation to place value.	Round a whole number to the nearest 10. Round a whole number to the nearest 100.	

STANDARD AND DECONSTRUCTION

EXPLANATIONS AND EXAMPLES

Students learn when and why to round numbers. They identify possible answers and halfway points. Then they narrow where the given number falls between the possible answers and halfway points. They also understand that by convention if a number is exactly at the halfway point of the two possible answers, the number is rounded up.

Example: Round 178 to the nearest 10.



STANDARD AND DECONSTRUCTION			
3.NBT.2	**Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.**		
DESCRIPTION	This standard refers to fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). The word "algorithm" refers to a procedure or a series of steps. There are other algorithms other than the standard algorithm. Third Grade students should have experiences beyond the standard algorithm. Computation Algorithm: a set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. Computation strategy: purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. (Progressions for the CCSSM; Number and Operation in Base Ten, CCSS Writing Team, April 2011, page 2)		
ESSENTIAL QUESTION(S) FOR THE STANDARD	How can understanding place value help you add or subtract in an efficient manner?		
MATHEMATICAL PRACTICE(S)	3.MP.2. Reason abstractly and quantitatively. 3.MP.7. Look for and make use of structure. 3.MP.8. Look for and express regularity in repeated reasoning.		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Know strategies and algorithms for adding and subtracting within 1000. Fluently add and subtract within 1000.		

EXPLANATIONS AND EXAMPLES

Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable. An interactive whiteboard or document camera may be used to show and share student thinking.

Example:

- Mary read 573 pages during her summer reading challenge. She was required to read only 399 pages. How many extra pages did Mary read beyond the challenge requirements?

Students may use several approaches to solve the problem including the traditional algorithm. Examples of other methods students may use are listed below:

- 399 + 1 = 400, 400 + 100 = 500, 500 + 73 = 573, therefore 1+ 100 + 73 = 174 pages (Adding up strategy)
- 400 + 100 is 500; 500 + 73 is 573; 100 + 73 is 173 plus 1 (for 399, to 400) is 174 (Compensating strategy)
- Take away 73 from 573 to get to 500, take away 100 to get to 400, and take away 1 to get to 399. Then 73 +100 + 1 = 174 (Subtracting to count down strategy)
- 399 + 1 is 400, 500 (that's 100 more). 510, 520, 530, 540, 550, 560, 570, (that's 70 more), 571, 572, 573 (that's 3 more) so the total is 1 + 100 + 70 + 3 = 174 (Adding by tens or hundreds strategy)

STANDARD AND DECONSTRUCTION

3.NBT.3	**Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.**
DESCRIPTION	This standard extends students' work in multiplication by having them apply their understanding of place value. This standard expects that students go beyond tricks that hinder understanding, such as "just adding zeros," and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens, and that twenty tens equals 200. The special role of 10 in the base-ten system is important in understanding multiplication of one-digit numbers with multiples of 10. For example, the product 3 x 50 can be represented as 3 groups of 5 tens, which is 15 tens, which is 150. This reasoning relies on the associative property of multiplication: 3 x 50 = 3 x (5 x 10) = (3 x 5) x 10 = 15 x 10 = 150. It is an example of how to explain an instance of a calculation pattern for these products: calculate the product of the non-zero digits, and then shift the product one place to the left to make the result ten times as large. • **Grade 3 explanations for "15 tens is 150"** • Skip-counting by 50. 5 tens is 50, 100, 150. • Counting on by 5 tens. 5 tens is 50, 5 more tens is 100, 5 more tens is 150. • Decomposing 15 tens. 15 tens is 10 tens and 5 tens. 10 tens is 100. 5 tens is 50. So 15 tens is 100 and 50, or 150. • Decomposing 15. $$\begin{aligned} 15 \times 10 &= (10+5) \times 10 \\ &= (10 \times 10) + (5 \times 10) \\ &= 100 + 50 \\ &= 150 \end{aligned}$$ *All of these explanations are correct. However, skip-counting and counting on become more difficult to use accurately as numbers become larger, e.g., in computing 5×90 or explaining why 45 tens is 450, and needs modification for products such as 4×90. The first does not indicate any place value understanding.* *(Progressions for the CCSSM; Number and Operation in Base Ten, CCSS Writing Team, April 2011, page 11)*
ESSENTIAL QUESTION(S) FOR THE STANDARD	How can understanding place value help you multiply in an efficient manner?
MATHEMATICAL PRACTICE(S)	3.MP.2. Reason abstractly and quantitatively. 3.MP.7. Look for and make use of structure. 3.MP.8. Look for and express regularity in repeated reasoning.

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Know strategies to multiply one-digit whole numbers by multiples of 10 (up to 90).	Apply knowledge of place value to multiply one-digit whole numbers by multiples of 10 in the range 10-90.	

EXPLANATIONS AND EXAMPLES

Students use base ten blocks, diagrams, or hundreds charts to multiply one-digit numbers by multiples of 10 from 10-90. They apply their understanding of multiplication and the meaning of the multiples of 10. For example, 30 is 3 tens and 70 is 7 tens. They can interpret 2 x 40 as 2 groups of 4 tens or 8 groups of ten. They understand that 5 x 60 is 5 groups of 6 tens or 30 tens and know that 30 tens is 300. After developing this understanding, they begin to recognize the patterns in multiplying by multiples of 10.

Students may use manipulatives, drawings, document camera, or interactive whiteboard to demonstrate their understanding.

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

DOMAIN:

NUMBER & OPERATIONS - FRACTIONS (NF)

THIRD GRADE

MATHEMATICS

DOMAINS	**Number and Operations—Fractions**
CLUSTERS	**1. Develop understanding of fractions as numbers.**

NUMBER & OPERATIONS IN BASE TEN (NBT)		
SECOND	**THIRD**	**FOURTH**
Fractions		
Section 1: Working with Unit Fractions	Section 1: Working with Unit Fractions	Section 1: Working with Unit Fractions
	3.NF.2.a Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.	
	3.NF.2.a Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.	
Section 2: Equivalence and Comparison of Fractions	Section 2: Equivalence and Comparison of Fractions	Section 2: Equivalence and Comparison of Fractions
	3.NF.3.c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.	4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
	3.NF.3.b Recognize and generate simple equivalent fractions, (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).	4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols <, =, or >, and justify the conclusions, e.g., by using a visual fraction model.
	3.NF.3.a Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.	
	3.NF.3.d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <, =, or >, and justify the conclusions, e.g., by using a visual fraction model.	
Section 3: Operations with Fractions	Section 3: Operations with Fractions	Section 3: Operations with Fractions
		4.NF.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

NUMBER & OPERATIONS IN BASE TEN (NBT)		
SECOND	**THIRD**	**FOURTH**
		4.NF.3.a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
		4.NF.3.c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
		4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.
		4.NF.3.d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Source: turnonccmath.net, NC State University College of Education

CLUSTER:	**1. Develop understanding of fractions as numbers.**
BIG IDEA:	Fractions are numbers that are part of a whole
ACADEMIC VOCABULARY:	partition(ed), equal parts, fraction, equal distance (intervals), equivalent, equivalence, reasonable, denominator, numerator, comparison, compare, ‹, ›, = , justify

STANDARD AND DECONSTRUCTION

3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

DESCRIPTION

This standard refers to the sharing of a whole being partitioned. Fraction models in Third Grade include only area (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a group) are not addressed in Third Grade.

In 3.NF.1 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and reasoning about one part of the whole, e.g., if a whole is partitioned into 4 equal parts then each part is 1⁄4 of the whole, and 4 copies of that part make the whole. Next, students build fractions from unit fractions, seeing the numerator 3 of 3⁄4 as saying that 3⁄4 is the quantity you get by putting 3 of the 1⁄4's together. There is no need to introduce "improper fractions" initially.

The importance of specifying the whole

Without specifying the whole it is not reasonable to ask what fraction is represented by the shaded area. If the left square is the whole, the shaded area represents the fraction $\frac{3}{2}$; if the entire rectangle is the whole, the shaded area represents $\frac{3}{4}$.

(Progressions for the CCSSM; Number and Operation – Fractions, CCSS Writing Team, August 2011, page 2)

Initially, students can use an intuitive notion of "same size and same shape" (congruence) to explain why the parts are equal, e.g., when they divide a square into four equal squares or four equal rectangles. Students come to understand a more precise meaning for "equal parts" as "parts with equal measurements." For example, when a ruler is partitioned into halves or quarters of an inch, they see that each subdivision has the same length. In area models they reason about the area of a shaded region to decide what fraction of the whole it represents.

Area representations of $\frac{1}{4}$

In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is $\frac{1}{4}$ of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.

(Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 3)

ESSENTIAL QUESTION(S) FOR THE STANDARD	How does a fraction differ from a whole number?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics 3.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Identify a fraction such as 2/3 and explain that the quantity formed is 2 equal parts of the whole partitioned into 3 equal parts (1/3 and 1/3 of the whole 3/3). Recognize a unit fraction such as 1/4 as the quantity formed when the whole is partitioned into 4 equal parts.	Express a fraction as the number of unit fractions; Use accumulated unit fractions to represent numbers equal to, less than, and greater than one (1/3 and 1/3 is 2/3; 1/3, 1/3, 1/3, and 1/3 is 4/3).	

EXPLANATIONS AND EXAMPLES

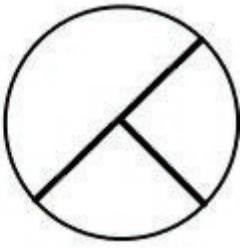
Some important concepts related to developing understanding of fractions include:

- Understand fractional parts must be equal-sized

Example

These are thirds

Non-example



These are NOT thirds

- The number of equal parts tell how many make a whole.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
- The size of the fractional part is relative to the whole.
- The number of children in one-half of a classroom is different than the number of children in one-half of a school. (The whole in each set is different; therefore, the half in each set will be different).
- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.
- ¾ means that there are 3 one-fourths.
- Students can count one fourth, two fourths, three fourths.

STANDARD AND DECONSTRUCTION

Students express fractions as fair sharing, parts of a whole, and parts of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing.

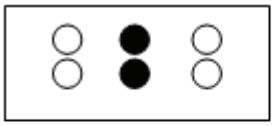
To develop understanding of fair shares, students first participate in situations where the number of objects is greater than the number of children and then progress into situations where the number of objects is less than the number of children.

Examples:

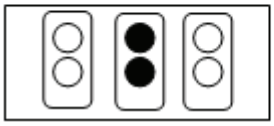
- Four children share six brownies so that each child receives a fair share. How many brownies will each child receive?
- Six children share four brownies so that each child receives a fair share. What portion of each brownie will each child receive?
- What fraction of the rectangle is shaded? How might you draw the rectangle in another way but with the same fraction shaded?

EXPLANATIONS AND EXAMPLES

What fraction of the set is black? Solution: $\frac{2}{4}$ or $\frac{1}{2}$

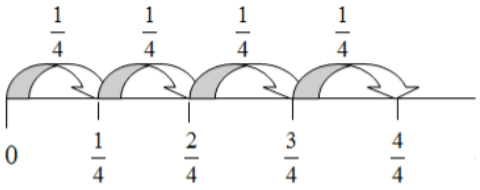
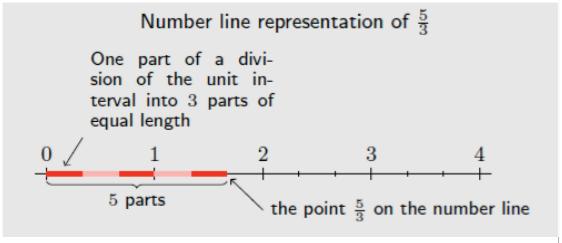


Solution: $\frac{2}{6}$



Solution: $\frac{1}{3}$

STANDARD AND DECONSTRUCTION

3.NF.2	**Understand a fraction as a number on the number line; represent fractions on a number line diagram.**
DESCRIPTION	The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that 1/2 is between 0 and 1). Students need ample experiences folding linear models (e.g., string, sentence strips) to help them reason about and justify the location of fractions, such that 1/2 lies exactly halfway between 0 and 1. In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or 1/4 (3.NF.2a). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction 3/4 (3.NF.2b).   (Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 3)
ESSENTIAL QUESTION(S) FOR THE STANDARD	How does a fraction differ from a whole number?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.7. Look for and make use of structure.

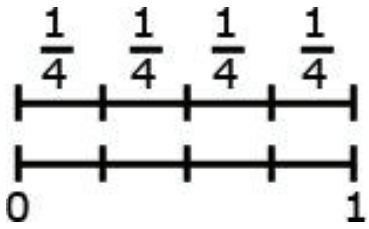
SUBSTANDARD DECONSTRUCTION:	**3.NF.2a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based on 0 locates the number 1/b on the number line.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Define the interval from 0 to 1 on a number line as the whole. Divide a whole on a number line into equal parts; Recognize that the equal parts between 0 and 1 have a fractional representation.	Explain that the end of each equal part is represented by a fraction (1/ the number of equal parts). Represent each equal part on a number line with a fraction.	

SUBSTANDARD DECONSTRUCTION:	**3.NF.2b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:		Explain that the endpoint of each equal part represents the total number of equal parts. Represent each equal part on a number line with a fraction.	

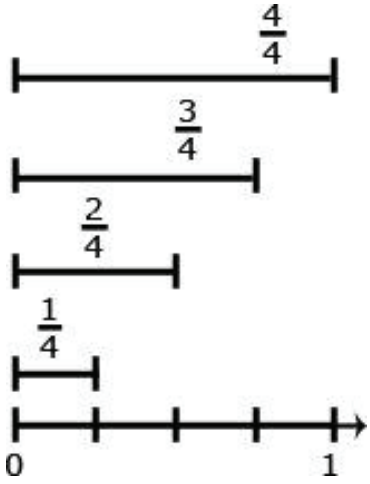
EXPLANATIONS AND EXAMPLES

Students transfer their understanding of parts of a whole to partition a number line into equal parts. There are two new concepts addressed in this standard which students should have time to develop.

1. On a number line from 0 to 1, students can partition (divide) it into equal parts and recognize that each segmented part represents the same length.



2. Students label each fractional part based on how far it is from zero to the endpoint.



An interactive whiteboard may be used to help students develop these concepts.

STANDARD AND DECONSTRUCTION

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

DESCRIPTION

3.NF.3a and 3.NF.3b These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.

This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of a/1.

Example:

If 6 brownies are shared between 2 people, how many brownies would each person get?

This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that 1/3 of a cake is larger than 1⁄4 of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.

In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example, 1⁄2 of a large pizza is a different amount than 1⁄2 of a small pizza. Students should be given opportunities to discuss and reason about which 1⁄2 is larger.

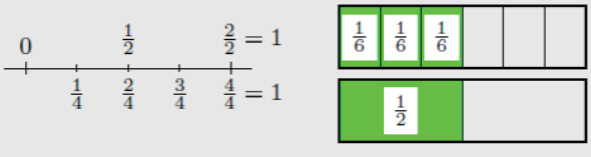
In Second Grade, students compared lengths using a standard measurement unit. In Third Grade they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions. For example, segment from 0 to 3⁄4 is shorter than the segment from 0 to 5/4 because it measures 3 units of 1⁄4 as opposed to 5 units of 1⁄4, therefore 3⁄4 < 5/4.

Students also see that for unit fractions, the fraction with the larger denominator is smaller. They can then reason that for more or identical pieces to make the same whole, the pieces must be smaller.

From this they can reason that if fractions have the same numerator, the fraction with the smallest denominator is greater. For example, 2/5 > 2/7 because 1/7 < 1/5, so 2 lengths of 1/7 is less than 2 lengths of 1/5.

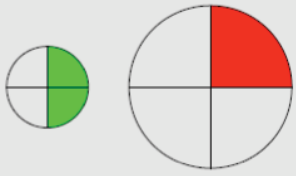
As with equivalence of fractions, it is important in comparing fractions to make sure that each fraction refers to the same whole.

Using the number line and fraction strips to see fraction equivalence



(Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 4)

The importance of referring to the same whole when comparing fractions



A student might think that $\frac{1}{4} > \frac{1}{2}$, because a fourth of the pizza on the right is bigger than a half of the pizza on the left.

(Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 4)

3rd Grade Mathematics ● Unpacked Content Page 26

ESSENTIAL QUESTION(S) FOR THE STANDARD	How can two fractions have the same value?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.2. Reason abstractly and quantitatively. 3.MP.3. Construct viable arguments and critique the reasoning of others. 3.MP.4. Model with mathematics. 3.MP.6. Attend to precision. 3.MP.7. Look for and make use of structure. 3.MP.8. Look for and express regularity in repeated reasoning.

SUBSTANDARD DECONSTRUCTION:	**3.NF.3a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Describe equivalent fractions. Recognize simple equivalent fractions .	Compare fractions by reasoning about their size to determine equivalence. Use number lines, size, visual fraction models, etc. to find equivalent fractions.	

SUBSTANDARD DECONSTRUCTION:	**3.NF.3b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Recognize whole numbers written in fractional parts on a number line. Recognize the difference between a whole number and a fraction.	Explain how a fraction is equivalent to a whole number.	

SUBSTANDARD DECONSTRUCTION:	**3.NF.3c Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.**

DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Tasks assessing concepts, skills, and procedures.	Tasks assessing expressing mathematical reasoning.	Tasks assessing modeling/application.

SUBSTANDARD DECONSTRUCTION:	**3.NF.3d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☒ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Recognize whether or not different fractions refer to the same whole. Determine if comparisons of fractions can be made (if they refer to the same whole). Compare two fractions with the same numerator by reasoning about their size. Compare two fractions with the same denominator by reasoning about their size. Record the results of comparisons using symbols >, =, or <. Justify conclusions about the equivalence of fractions. Explain what the numerator in a fraction represents and its location. Explain what the denominator in a fraction represents and its location.		

EXPLANATIONS AND EXAMPLES

An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, 1/8 is smaller than 1/2 because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces. Students recognize when examining fractions with common denominators, the wholes have been divided into the same number of equal parts. So the fraction with the larger numerator has the larger number of equal parts.

2/6 < 5/6

To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts are different. They can infer that the same number of smaller pieces is less than the same number of bigger pieces.

3/8 < 3/4

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

DOMAIN:

MEASUREMENT AND DATA (MD)

THIRD GRADE

MATHEMATICS

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

DOMAIN	**Measurement and Data**
CLUSTERS	**1. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** **2. Represent and interpret data.** **3. Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** **4. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

MEASUREMENT AND DATA (MD)		
SECOND	**THIRD**	**FOURTH**
Time and Money		
Section 1: Time	Section 1: Time	Section 1: Time
2.MD.7 Read and write time (digital and analog) to nearest 5 minutes.	3.MD.1 Read and write time to nearest minute and calculate time intervals.	
Section 2: Money	Section 2: Money	Section 2: Money
2.MD.8 Solve word problems involving money (dollars, quarters, dimes, nickels, and pennies) including symbols.		
Length, Area, and Volume		
Section 2: Length Measurement using Units and Tools	Section 2: Length Measurement using Units and Tools	Section 2: Length Measurement using Units and Tools
2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.		
2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.		
2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.		
2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.		
2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.		
2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole-number sums and differences within 100 on a number line diagram.		
Section 3: Area and Perimeter	Section 3: Area and Perimeter	Section 3: Area and Perimeter
	3.MD.5.b A plane figure that can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	4.MD.3 Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.
	3.MD.5.a A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.	
	3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).	

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

MEASUREMENT AND DATA (MD)		
SECOND	**THIRD**	**FOURTH**
	3.MD.7.a Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	
	3.MD.7.b Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	
	3.MD.7.c Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.	
	3.MD.7.d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	
	3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	
	3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes compound units such as cm3 and finding the geometric volume of a container.)	
Section 5: Conversion	Section 5: Conversion	Section 5: Conversion
		4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.
		4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

MEASUREMENT AND DATA (MD)		
SECOND	**THIRD**	**FOURTH**
Elementary Data and Monitoring		
Section 2: Modeling with Data	Section 2: Modeling with Data	Section 2: Modeling with Data
2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.	4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.	3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters.	

Source: turnonccmath.net, NC State University College of Education

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

CLUSTER:	**1. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**
BIG IDEA:	Measurement and estimation help with daily tasks (time, volume, mass).
ACADEMIC VOCABULARY:	estimate, time, time intervals, minute, hour, elapsed time, measure, liquid, volume, mass, standard units, metric, gram (g), kilogram (kg), liter (l)

STANDARD AND DECONSTRUCTION

3.MD.1	**Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.**
DESCRIPTION	This standard calls for students to solve elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students).
ESSENTIAL QUESTION(S) FOR THE STANDARD	Why is telling time to the minute important?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.6. Attend to precision.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Recognize minute marks on an analog clock face and minute position on a digital clock face. Know how to write time to the minute.	Compare an analog clock face with a number line diagram. Use a number line diagram to add and subtract time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes.	Tell time to the minute.

EXPLANATIONS AND EXAMPLES	Students in Second Grade learned to tell time to the nearest five minutes. In Third Grade, they extend telling time and measure elapsed time both in and out of context using clocks and number lines. Students may use an interactive whiteboard to demonstrate understanding and justify their thinking.

STANDARD AND DECONSTRUCTION

3.MD.2

Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

DESCRIPTION

This standard asks for students to reason about the units of mass and volume using units g, kg, and l. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter emphasizing the relationship between smaller units to larger units in the same system. Word problems should only be one-step and include the same units.

Students are not expected to do conversions between units, but reason as they estimate, using benchmarks to measure weight and capacity.

Example:

Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram.

Example:

A paper clip weighs about a) a gram, b) 10 grams, c) 100 grams? Explain why.

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).
- Understand the relationship between the size of a unit and the number of units needed (compensatory principal).

Before learning to measure attributes, children need to recognize them, distinguishing them from other attributes. That is, the attribute to be measured has to "stand out" for the student and be discriminated from the undifferentiated sense of amount that young children often have, labeling greater lengths, areas, volumes, and so forth, as "big" or "bigger."

These standards do not differentiate between weight and mass. Technically, mass is the amount of matter in an object. Weight is the force exerted on the body by gravity. On the earth's surface, the distinction is not important; on the moon, an object would have the same mass would weigh less due to the lower gravity.

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 2)

Much of the work involving measure support the work the emphasized in third on multiplication. Example: number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students).

Table 1: Multiplication and division situations for measurement

	Unknown Product A × B = ☐	Group Size Unknown A × ☐ = C and C ÷ A = ☐	Number of Groups Unknown ☐ × B = C and C ÷ B = ☐
Grouped Objects (Units of Units)	You need A lengths of string, each B inches long. How much string will you need altogether?	You have C inches of string, which you will cut into A equal pieces. How long will each piece of string be?	You have C inches of string, which you will cut into pieces that are B inches long. How many pieces of string will you have?
Arrays of Objects (Spatial Structuring)	What is the area of a A cm by B cm rectangle?	A rectangle has area C square centimeters. If one side is A cm long, how long is a side next to it?	A rectangle has area C square centimeters. If one side is B cm long, how long is a side next to it?
Compare	A rubber band is B cm long. How long will the rubber band be when it is stretched to be A times as long?	A rubber band is stretched to be C cm long and that is A times as long as it was at first. How long was the rubber band at first?	A rubber band was B cm long at first. Now it is stretched to be C cm long. How many times as long is the rubber band now as it was at first?

Adapted from box 2-4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33. Note that Grade 3 work does not include Compare problems with "times as much," see the Operations and Algebraic Thinking Progression, Table 3, also p. 29.

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 19)

ESSENTIAL QUESTION(S) FOR THE STANDARD	Why is telling time to the minute important?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.2. Reason abstractly and quantitatively, 3.MP.4. Model with mathematics. 3.MP.5. Use appropriate tools strategically. 3.MP.6. Attend to precision.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Explain how to measure liquid volume in liters. Explain how to measure mass in grams and kilograms. Know various strategies to represent a word problem involving liquid volume or mass. Add, subtract, multiply, and divide units of liters, grams, and kilograms.	Solve one-step word problems involving masses given in the same units. Solve one-step word problems involving liquid volume given in the same units.	Measure liquid volumes using standard units of liters. Measure mass of objects using standard units of grams (g) and kilograms (kg).

EXPLANATIONS AND EXAMPLES	Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter. Example: Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) weighs about 100 grams so 10 boxes would weigh one kilogram.

CLUSTER:	**2. Represent and interpret data.**
BIG IDEA:	Graphs and Plots are used to represent measured and estimated plots.
ACADEMIC VOCABULARY:	scale, scaled picture graph, scaled bar graph, line plot, data

3.MD.3	**Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.**
DESCRIPTION	While exploring data concepts, students should Pose a question, Collect data, Analyze data, and Interpret data (PCAI). Students should be graphing data that is relevant to their lives Example: Pose a question: Student should come up with a question. What is the typical genre read in our class? Collect and organize data: student survey
ESSENTIAL QUESTION(S) FOR THE STANDARD	Why is accuracy important when drawing a graph or line plot?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.6. Attend to precision. 3.MP.7. Look for and make use of pattern.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

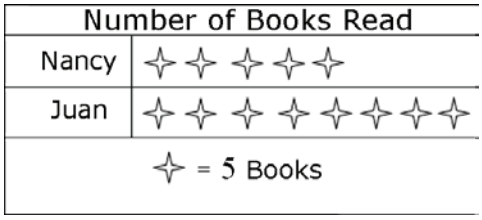
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Explain the scale of a graph with a scale greater than one. Identify the scale of a graph with a scale greater than one.	Analyze a graph with a scale greater than one. Choose a proper scale for a bar graph or picture graph. Interpret a bar/picture graph to solve one- or two-step problems asking "how many more" and "how many less".	Create a scaled picture graph to show data. Create a scaled bar graph to show data.

STANDARD AND DECONSTRUCTION

EXPLANATIONS AND EXAMPLES

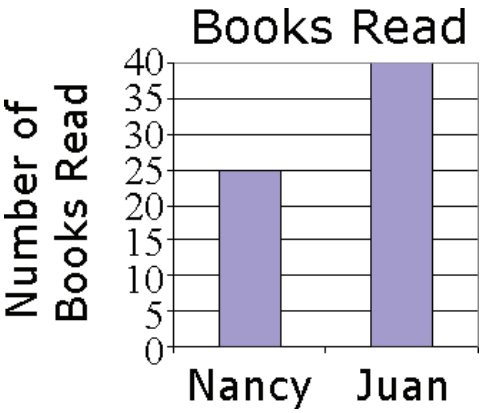
Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts.

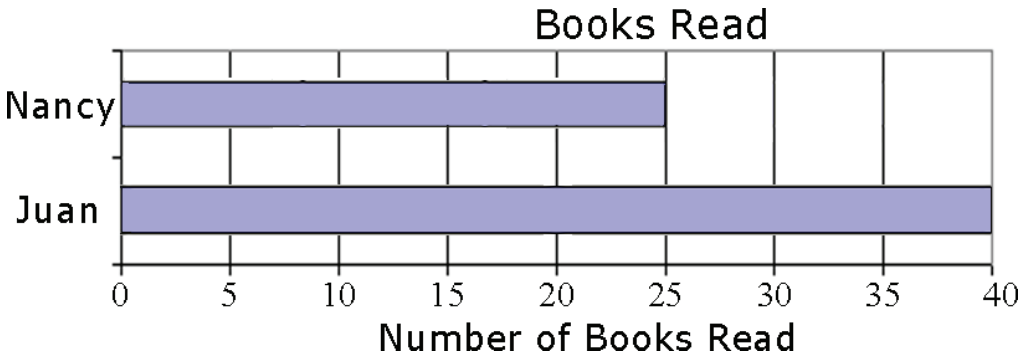
- Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data.



How many more books did Juan read than Nancy?

- Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.





STANDARD AND DECONSTRUCTION

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

DESCRIPTION

In Third Grade, students are beginning to learn fraction concepts (3.NF). They understand fraction equivalence in simple cases, and they use visual fraction models to represent and order fractions. Grade 3 students also measure lengths using rulers marked with halves and fourths of an inch. They use their developing knowledge of fractions and number lines to extend their work from the previous grade by working with measurement data involving fractional measurement values.

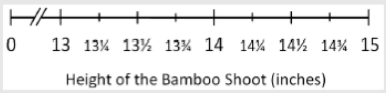
For example, every student in the class might measure the height of a bamboo shoot growing in the classroom, leading to the data set shown in the table. (Illustration below shows a larger data set than students would normally work with in elementary grades.)

To make a line plot from the data in the table, the student can determine the greatest and least values in the data: 13 1/2 inches and 14 3/4 inches. The student can draw a segment of a number line diagram that includes these extremes, with tick marks indicating specific values on the measurement scale. This is just like part of the scale on a ruler. Having drawn the number line diagram, the student can proceed through the data set recording each observation by drawing a symbol, such as a dot, above the proper tick mark. As with Second Grade line plots, if a particular data value appears many times in the data set, dots will "pile up" above that value. There is no need to sort the observations or to do any counting of them before producing the line plot. Students can pose questions about data presented in line plots, such as how many students obtained measurements larger than 14 1/4 inches.

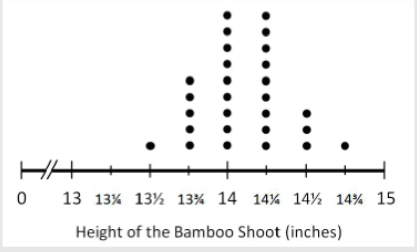
Students' measurements of a statue and of a bamboo shoot

Statue measurements		Bamboo shoot measurements	
Student's initials	Student's measured value (inches)	Student's initials	Height value (inches)
W.B.	64	W.B.	13 3/4
D.W.	65	D.W.	14 1/2
H.D.	65	H.D.	14 1/4
G.W.	65	G.W.	14 3/4
V.Y.	67	V.Y.	14 1/4
T.T.	66	T.T.	14 1/2
D.F.	67	D.F.	14
B.H.	65	B.H.	13 1/2
H.H.	63	H.H.	14 1/4
V.H.	64	V.H.	14 1/4
I.O.	64	I.O.	14 1/4
W.N.	65	W.N.	14
B.P.	69	B.P.	14 1/2
V.A.	65	V.A.	13 3/4
H.L.	66	H.L.	14
O.M.	64	O.M.	13 3/4
L.E.	65	L.E.	14 1/4
M.J.	66	M.J.	13 3/4
T.D.	66	T.D.	14 1/4
K.P.	64	K.P.	14
H.N.	65	H.N.	14
W.M.	67	W.M.	14
C.Z.	64	C.Z.	13 3/4
J.I.	66	J.I.	14
M.S.	66	M.S.	14 1/4
T.C.	65	T.C.	14
G.V.	67	G.V.	14
O.F.	65	O.F.	14 1/4

A scale for a line plot of the bamboo shoot data



A line plot of the bamboo shoot data



(Progressions for the CCSSM, Measurement Data, CCSS Writing Team, June 2011, page 10)

ESSENTIAL QUESTION(S) FOR THE STANDARD	Why is accuracy important when drawing a graph or line plot?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.6. Attend to precision.
DOK Range Target for Instruction & Assessment	☐ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Define horizontal axis. Identify each plot on the line as data or a number of objects.	Analyze data from a line plot. Determine appropriate unit of measurement. Determine appropriate scale for line plot.	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Create a line plot where the horizontal scale is marked off in appropriate units - whole numbers, halves, or quarters.

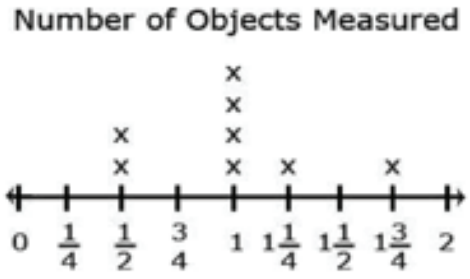
EXPLANATIONS AND EXAMPLES

Students in Second Grade measured length in whole units using both metric and U.S. customary systems. It's important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third Graders need many opportunities measuring the length of various objects in their environment.

Some important ideas related to measuring with a ruler are:

- The starting point of where one places a ruler to begin measuring.
- Measuring is approximate. Items that students measure will not always measure exactly ¼, ½ or one whole inch. Students will need to decide on an appropriate estimate length.
- Making paper rulers and folding to find the half and quarter marks will help students develop a stronger understanding of measuring length.

Students generate data by measuring and create a line plot to display their findings. An example of a line plot is shown below:



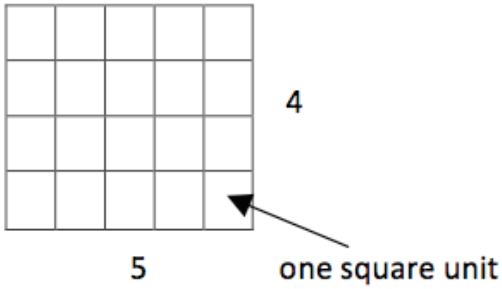
CLUSTER:	**3. Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**
BIG IDEA:	What we measure influences how we measure.
ACADEMIC VOCABULARY:	attribute, area, square unit, plane figure, gap, overlap, square cm, square m, square in, square ft, nonstandard units, tiling, side length, decomposing

STANDARD AND DECONSTRUCTION

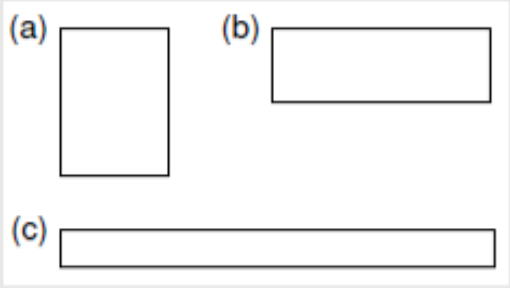
3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

DESCRIPTION

These standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper. Based on students' development, they should have ample experiences filling a region with square tiles before transitioning to pictorial representations on graph paper.



Which rectangle covers the most area?



These rectangles are formed from unit squares (tiles students have used) although students are not informed of this or the rectangle's dimensions: (a) 4 by 3, (b) 2 by 6, and (c) 1 row of 12. Activity from Lehrer, et al., 1998, "Developing understanding of geometry and space in the primary grades," in R. Lehrer & D. Chazan (Eds.), *Designing Learning Environments for Developing Understanding of Geometry and Space*, Lawrence Erlbaum Associates.

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 16)

ESSENTIAL QUESTION(S) FOR THE STANDARD	What does the area of a 2 dimensional figure represent?
MATHEMATICAL PRACTICE(S)	3.MP.2. Reason abstractly and quantitatively. 3.MP.4. Model with mathematics. 3.MP.5. Use appropriate tools strategically. 3.MP.6. Attend to precision.

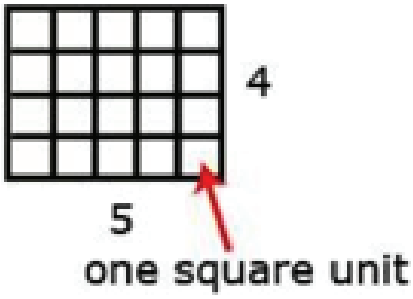
SUBSTANDARD DECONSTRUCTION:	**3.MD.5a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Define unit square . Define area.		

SUBSTANDARD DECONSTRUCTION:	**3.MD.5b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:		Relate the number (n) of unit squares to the area of a plane figure.	Relate the number (n) of unit squares to the area of a plane figure.

EXPLANATIONS AND EXAMPLES

Students develop understanding of using square units to measure area by:

- Using different sized square units.
- Filling in an area with the same sized square units and counting the number of square units.
- An interactive whiteboard would allow students to see that square units can be used to cover a plane figure.



STANDARD AND DECONSTRUCTION

3.MD.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
DESCRIPTION	Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.
ESSENTIAL QUESTION(S) FOR THE STANDARD	What does the area of a 2 dimensional figure represent?
MATHEMATICAL PRACTICE(S)	3.MP.5. Use appropriate tools strategically. 3.MP.6. Attend to precision.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	Know: Concepts/Skills	Think	Do
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Measure areas by counting unit squares. Use unit squares of cm, m, in, ft, and other sizes of unit squares to measure area.		

EXPLANATIONS AND EXAMPLES	Using different sized graph paper, students can explore the areas measured in square centimeters and square inches. An interactive whiteboard may also be used to display and count the unit squares (area) of a figure.

STANDARD AND DECONSTRUCTION

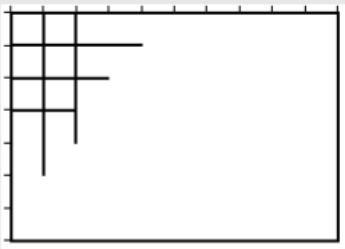
3.MD.7 Relate area to the operations of multiplication and addition.

DESCRIPTION

Students can learn how to multiply length measurements to find the area of a rectangular region. But in order that they make sense of these quantities, they must first learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows. This relies on the development of spatial structuring. To build from spatial structuring to understanding the number of area-units as the product of number of units in a row and number of rows, students might draw rectangular arrays of squares and learn to determine the number of squares in each row with increasingly sophisticated strategies, such as skip-counting the number in each row and eventually multiplying the number in each row by the number of rows. They learn to partition a rectangle into identical squares by anticipating the final structure and forming the array by drawing line segments to form rows and columns. They use skip counting and multiplication to determine the number of squares in the array.

Many activities that involve seeing and making arrays of squares to form a rectangle might be needed to build robust conceptions of a rectangular area structured into squares.

Incomplete array



To determine the area of this rectangular region, students might be encouraged to construct a row, corresponding to the indicated positions, then repeating that row to fill the region. Cutouts of strips of rows can help the needed spatial structuring and reduce the time needed to show a rectangle as rows or columns of squares. Drawing all of the squares can also be helpful, but it is slow for larger rectangles. Drawing the unit lengths on the opposite sides can help students see that joining opposite unit end-points will create the needed unit square grid.

Students should understand and explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle's interior. For example, students might explain that one length tells how many unit squares in a row and the other length tells how many rows there are.

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 17)

Students should tile rectangle then multiply the side lengths to show it is the same.

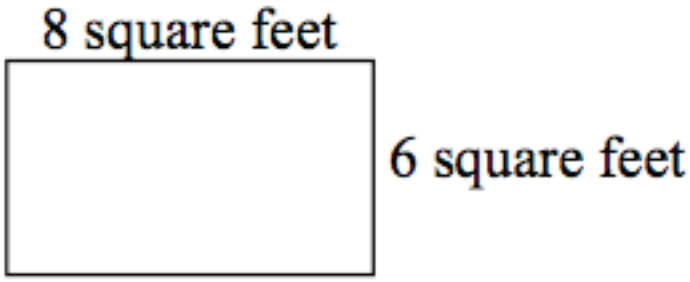
To find the area one could count the squares or multiply 3 x 4 = 12.

1	2	3	4
5	6	7	8
9	10	11	12

Students should solve real world and mathematical problems.

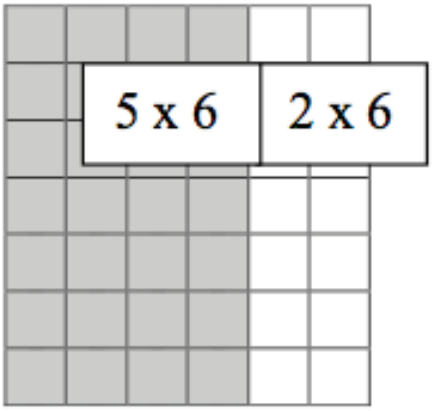
Example: Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?

DESCRIPTION CONTINUED



Students might solve problems such as finding all the rectangular regions with whole-number side lengths that have an area of 12 area-units, doing this for larger rectangles (e.g., enclosing 24, 48, 72 area-units), making sketches rather than drawing each square. Students learn to justify their belief they have found all possible solutions. (Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 18)

This standard extends students' work with the distributive property. For example, in the picture below the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.



Using concrete objects or drawings students build competence with composition and decomposition of shapes, spatial structuring, and addition of area measurements, students learn to investigate arithmetic properties using area models. For example, they learn to rotate rectangular arrays physically and mentally, understanding that their areas are preserved under rotation, and thus, for example, 4 x 7 = 7 x 4, illustrating the commutative property of multiplication. Students also learn to understand and explain that the area of a rectangular region of, for example, 12 length-units by 5 length-units can be found either by multiplying 12 x 5, or by adding two products, e.g., 10 x 5 and 2 x 5, illustrating the distributive property. (Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 18)

With strong and distinct concepts of both perimeter and area established, students can work on problems to differentiate their measures. For example, they can find and sketch rectangles with the same perimeter and different areas or with the same area and different perimeters and justify their claims. Differentiating perimeter from area is facilitated by having students draw congruent rectangles and measure, mark off, and label the unit lengths all around the perimeter on one rectangle, then do the same on the other rectangle but also draw the square units. This enables students to see the units involved in length and area and find patterns in finding the lengths and areas of non-square and square rectangles. Students can continue to describe and show the units involved in perimeter and area after they no longer need these (Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 18).

ESSENTIAL QUESTION(S) FOR THE STANDARD	What is an efficient strategy to determine the area of a 2 dimensional figure?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.2. Reason abstractly and quantitatively. 3.MP.4. Model with mathematics. 3.MP.5. Use appropriate tools strategically. 3.MP.6. Attend to precision.

SUBSTANDARD DECONSTRUCTION:	**3.MD.7a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Find the area of a rectangle by tiling it in unit squares. Find the side lengths of a rectangle in units.	Compare the area found by tiling a rectangle to the area found by multiplying the side lengths.	

SUBSTANDARD DECONSTRUCTION:	**3.MD.7b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.**		
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Multiply side lengths to find areas of rectangles.	Solve real world and mathematical area problems by multiplying side lengths of rectangles. Use rectangular arrays to represent whole-number products in multiplication problems.	

SUBSTANDARD DECONSTRUCTION:	**3.MD.7c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.**
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Multiply using an area model (array).	Relate area of a rectangle to multiplication and addition by modeling the distributive property.	

SUBSTANDARD DECONSTRUCTION:	**3.MD.7d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.**
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Recognize that areas of each rectangle in a rectilinear (straight line) figure can be added together to find the area of the figure. Find areas of rectangles Add areas of rectangles.	Use the technique of decomposing rectilinear figures to find the area of each rectangle to solve real world problems.	Decompose rectilinear figures into non-overlapping rectangles.

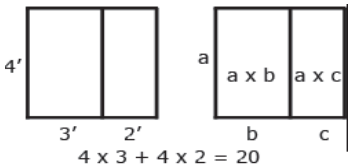
EXPLANATIONS AND EXAMPLES

Students tile areas of rectangles, determine the area, record the length and width of the rectangle, investigate the patterns in the numbers, and discover that the area is the length times the width.

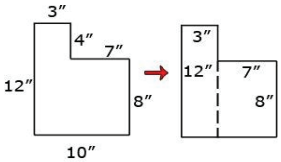
Example:

Joe and John made a poster that was 4′ by 3′. Mary and Amir made a poster that was 4′ by 2′. They placed their posters on the wall side-by-side so that that there was no space between them. How much area will the two posters cover?

Students use pictures, words, and numbers to explain their understanding of the distributive property in this context.



4 x 3 + 4 x 2 = 20
4 (3 + 2) = 20
4 x 5 = 20



area is 12 x 3 + 8 x 7 =
92 sq inches

Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.

CLUSTER:	**4. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**
BIG IDEA:	What we measure influences how we measure?
ACADEMIC VOCABULARY:	attribute, perimeter, plane figure, linear, area, polygon, side length

STANDARD AND DECONSTRUCTION

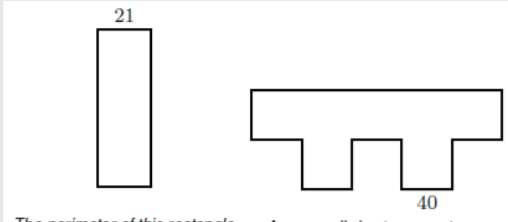
3.MD.8

Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

DESCRIPTION

A perimeter is the boundary of a two-dimensional shape. For a polygon, the length of the perimeter is the sum of the lengths of the sides. Initially, it is useful to have sides marked with unit length marks, allowing students to count the unit lengths. Later, the lengths of the sides can be labeled with numerals. As with all length tasks, students need to count the length-units and not the end-points. Next, students learn to mark off unit lengths with a ruler and label the length of each side of the polygon. For rectangles, parallelograms, and regular polygons, students can discuss and justify faster ways to find the perimeter length than just adding all of the lengths. Rectangles and parallelograms have opposite sides of equal length, so students can double the lengths of adjacent sides and add those numbers or add lengths of two adjacent sides and double that number. A regular polygon has all sides of equal length, so its perimeter length is the product of one side length and the number of sides. Perimeter problems for rectangles and parallelograms often give only the lengths of two adjacent sides or only show numbers for these sides in a drawing of the shape. The common error is to add just those two numbers. Having students first label the lengths of the other two sides as a reminder is helpful. Students then find unknown side lengths in more difficult "missing measurements" problems and other types of perimeter problems. (Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 16)

Missing measurements and other perimeter problems



The perimeter of this rectangle is 168 length units. What are the lengths of the three unlabeled sides?

Assume all short segments are the same length and all angles are right

Compare these problems with the "missing measurements" problems of Grade 2.

Another type of perimeter problem is to draw a robot on squared grid paper that meets specific criteria. All the robot's body parts must be rectangles. The perimeter of the head might be 36 length-units, the body, 72; each arm, 24; and each leg, 72. Students are asked to provide a convincing argument that their robots meet these criteria (MP3). Next, students are asked to figure out the area of each of their body parts (in square units). These are discussed, with students led to reflect on the different areas that may be produced with rectangles of the same perimeter. These types of problems can be also presented as turtle geometry problems. Students create the commands on paper and then give their commands to the Logo turtle to check their calculations. For turtle length units, the perimeter of the head might be 300 length-units, the body, 600; each arm, 400; and each leg, 640.

(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, June 2012, page 16)

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

ESSENTIAL QUESTION(S) FOR THE STANDARD	What does the perimeter of a polygon (2 dimensional figure) represent? What type of problem would require the use of area and perimeter as a solution?
MATHEMATICAL PRACTICE(S)	3.MP.1. Make sense of problems and persevere in solving them. 3.MP.4. Model with mathematics. 3.MP.7. Look for and make use of structure.
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4

Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Define a polygon. Define perimeter.	Find the perimeter when given the length of sides. Find the perimeter when there is an unknown side length.	Exhibit (design, create, draw, model, etc.) rectangles with the same perimeter and different areas. Exhibit rectangles with the same area and different perimeters

EXPLANATIONS AND EXAMPLES

Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects, use addition to find perimeters, and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles.

Students use geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.

Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard.

Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.

DOMAIN:

GEOMETRY (G)

THIRD GRADE

MATHEMATICS

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

DOMAIN	**Geometry (G)**
CLUSTERS	**1. Reason with shapes and their attributes.**

GEOMETRTY (G)		
SECOND	**THIRD**	**FOURTH**
	Equapartitioning	
Section 1: Equipartitioning Wholes	Section 1: Equipartitioning Wholes	Section 1: Equipartitioning Wholes
	3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.	
2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	
Section 1: Shapes and Properties	Section 1: Shapes and Properties	Section 1: Shapes and Properties
2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
Section 3: Angles	Section 3: AngGEOMETRYles	Section 3: Angles
		4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
		4.MD.5.a An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle" and can be used to measure angles.
		MD.5.b Define an n-degree angle as n 1-degree angles.
		4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
		4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

GEOMETRTY (G)		
SECOND	**THIRD**	**FOURTH**
Section 4: Symmetry	Section 4: Symmetry	Section 4: Symmetry
		4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Source: turnonccmath.net, NC State University College of Education

CLUSTER:	**1. Reason with shapes and their attributes.**
BIG IDEA:	Shapes describe patterns and reason in the physical world.
ACADEMIC VOCABULARY:	properties1, attributes1, features1, quadrilateral, open figure, closed figure , three-sided, 2-dimensional, rhombi, rectangles, and squares are subcategories of quadrilaterals, polygon, rhombus/rhombi, rectangle, square, partition, unit fraction, kite. From previous grades: triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere

STANDARD AND DECONSTRUCTION

3.G.1	**Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.**
DESCRIPTION	Third Grade students are expected to understand some re lationships that exist among plane shapes. They need to develop an understanding of the organization of categories and subcategories of common shapes. Teachers should provide numerous opportunities for students to sort shapes into recognized categories and discuss the attributes that define those categories. Guide students to discover that some shapes may belong to multiple categories. The following chart may help students to see how the categories go from general to more specific.

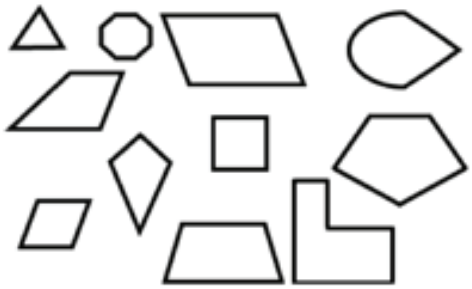
DESCRIPTION	Third Graders may benefit from participating in a discussion of other familiar general/specific categories as an analogy to help them grasp this concept. For example, lead a discussion about the following statement: "All jeans are pants, but not all pants are jeans." Ask students to agree or disagree and defend their answers. Further develop the concept by providing other pairs and asking students to create a statement that defines their relationship. For example, dogs/animals, carrots/vegetables, vowels/letters, etc. Once students have grasped the concept of comparing categories from general to specific, introduce shape relationships in the discussion.

ESSENTIAL QUESTION(S) FOR THE STANDARD	Why do we sort and classify shapes?
MATHEMATICAL PRACTICE(S)	3.MP.5. Use appropriate tools strategically. 3.MP.6. Attend to precision. 3.MP.7. Look for and make use of structure.

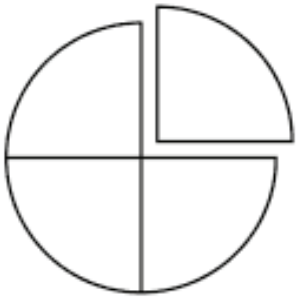
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Identify and define rhombuses, rectangles, and squares as examples of quadrilaterals based on their attributes.	Describe, analyze, and compare properties of two-dimensional shapes. Compare and classify shapes by attributes, sides, and angles Group shapes with shared attributes to define a larger category (e.g., quadrilaterals).	Draw examples of quadrilaterals that do and do not belong to any of the subcategories.

EXPLANATIONS AND EXAMPLES

In Second Grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third Graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.



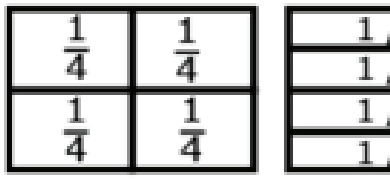
STANDARD AND DECONSTRUCTION

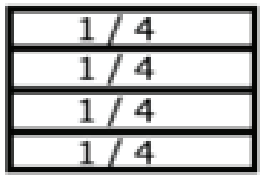
3.G.2	**Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.**
DESCRIPTION	In Third Grade, students start to develop the idea of a fraction more formally, building on the idea of partitioning a whole into equal parts. The whole can be a shape such as a circle or rectangle. In Grade 4, this is extended to include wholes that are collections of objects. This standard also builds on students' work with fractions and area. Students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths. Example: This figure was partitioned/divided into four equal parts. Each part is 1⁄4 of the total area of the figure. 
ESSENTIAL QUESTION(S) FOR THE STANDARD	Why do we sort and classify shapes?
MATHEMATICAL PRACTICE(S)	3.MP.2. Reason abstractly and quantitatively. 3.MP. 4. Model with mathematics. 3.MP.5. Use appropriate tools strategically.

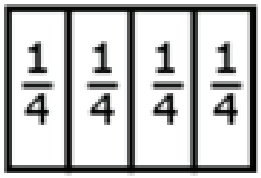
DOK Range Target for Instruction & Assessment	☒ 1 ☒ 2 ☐ 3 ☐ 4		
Instructional Targets	**Know: Concepts/Skills**	**Think**	**Do**
Instructional Targets	Tasks assessing concepts, skills and procedures	Tasks assessing expressing mathematical reasoning	Tasks assessing modeling applications
Students should be able to:	Know that shapes can be partitioned into equal areas. Describe the area of each part as a fractional part of the whole.	Relate fractions to geometry by expressing the area of part of a shape as a unit fraction of the whole.	Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.

EXPLANATIONS AND EXAMPLES

Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.







THIRD GRADE

LEXILE GRADE LEVEL BAND: 520L TO 820L

COMMON CORE State Standards

DECONSTRUCTED for CLASSROOM IMPACT

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